

LOGARITMOS

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A. INTRODUCCIÓN TEÓRICA

A.1 Definición de logaritmo:

Sea x un número. El logaritmo de ese número es el exponente al que hay que elevar cierta base b para obtener x :

$$x = b^y \Leftrightarrow y = \log_b x$$

Ejemplo:

- El logaritmo de 16 en base 2 es el exponente al que hay que elevar la base 2 para obtener 16, es decir, cuatro:

$$\log_2 16 = 4, \text{ ya que } 16 = 2^y \Leftrightarrow y = \log_2 16 = 4$$

A.2 Logaritmos naturales:

Los logaritmos que tienen como base al número e, son llamados “logaritmos naturales”. Se simbolizan con la abreviatura **ln**.

$$\ln x = \log_e x$$

A.3 Cambio de base en los logaritmos:

Si queremos expresar $\log_a x$ mediante $\log_b x$ sólo tenemos que tener en cuenta que:

$$\log_b M = \frac{\log_a M}{\log_a b}$$

A.4 Propiedades:

I. $\log_a MN = \log_a M + \log_a N$	IV. $\log_a 1 = 0$
II. $\log_a M^p = p \cdot \log_a M$	V. $\log_a a = 1$
III. $\log_a \frac{M}{N} = \log_a M - \log_a N$	VI. $a^{\log_a b} = b$

B. EJERCICIOS RESUELTOS

B.1. Dado un logaritmo, halla su valor:

$$1. \quad \log_2 64 = \log_2 2^6 = 6 \cdot \log_2 2 = 6 \cdot 1 = 6$$

$$2. \quad \log_2 \sqrt{2} = \log_2 2^{\frac{1}{2}} = \frac{1}{2} \cdot \log_2 2 = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

$$3. \quad \log_{\frac{1}{2}} \sqrt{2} = \log_{\frac{1}{2}} 2^{\frac{1}{2}} = \frac{1}{2} \cdot \log_{\frac{1}{2}} 2 = \frac{1}{2} \cdot \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{-1} = \frac{1}{2} \cdot (-1) \log_{\frac{1}{2}} \left(\frac{1}{2}\right) = -\frac{1}{2}$$

$$4. \quad \log_{\frac{1}{3}} \sqrt[5]{81} = \log_{\frac{1}{3}} \sqrt[5]{3^4} = \log_{\frac{1}{3}} (3^4)^{\frac{1}{5}} = \log_{\frac{1}{3}} 3^{\frac{4}{5}} = \frac{4}{5} \cdot \log_{\frac{1}{3}} 3 = \frac{4}{5} \cdot \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-1} = \\ = \frac{4}{5} \cdot (-1) \cdot \log_{\frac{1}{3}} \left(\frac{1}{3}\right) = -\frac{4}{5} \cdot 1 = -\frac{4}{5}$$

5. $\log_{10} (5 \log_{10} 100)^2 = 2 \log_{10} (5 \log_{10} 100) = 2 \log_{10} (5 \log_{10} 10^2) =$
 $= 2 \log_{10} (5 \cdot 2 \log_{10} 10) = 2 \log_{10} 10 = 2$
6. $\log_{\sqrt{2}} 32 = \log_{2^{\frac{1}{2}}} 2^5 = \log_{2^{\frac{1}{2}}} \left(2^{\frac{1}{2}} \right)^{10} = 10 \cdot \log_{2^{\frac{1}{2}}} \left(2^{\frac{1}{2}} \right) = 10$
7. $\log_{9\sqrt{3}} 3 \cdot \sqrt[5]{27} = \log_{3 \cdot 3^{\frac{1}{2}}} 3 \cdot (3^3)^{\frac{1}{5}} = \log_{3^{\frac{3}{2}+1}} 3^{\frac{3}{5}+1} = \log_{3^{\frac{5}{2}}} 3^{\frac{8}{5}} = \log_{3^{\frac{5}{2}}} \left(3^{\frac{3}{5}} \right)^{\frac{8}{3}} =$
 $= \frac{16}{15} \log_{3^{\frac{5}{2}}} \left(3^{\frac{3}{5}} \right) = \frac{16}{15}$

B.2. Dada una expresión logarítmica, hallar su valor.

8. $\log_2 \sqrt[5]{2} + \log_2 8 + \log_2 \frac{1}{4} = \log_2 2^{\frac{1}{5}} + \log_2 2^3 + \log_2 2^{-2} =$
 $= \frac{1}{5} \log_2 2 + 3 \log_2 2 - 2 \log_2 2 = \frac{1}{5} + 3 - 2 = \frac{6}{5}$
9. $\ln 1 + \ln e + \ln e^3 + \ln \sqrt[3]{e} + \ln \frac{1}{e} = 0 + 1 + 3 \ln e + \ln e^{\frac{1}{3}} + \ln e^{-1} =$
10. $\log 810 + \log 0,03 + \log \sqrt[5]{\frac{1}{9}}, \text{ si } \log 3 \approx 0,477$
 $\log 810 + \log 0,03 + \log \sqrt[5]{\frac{1}{9}} = \log (3^4 \cdot 10) + \log \frac{3}{100} + \log (3^{-2})^{\frac{1}{5}} =$
 $= 4 \log 3 + \log 10 + \log 3 - \log 10^2 - \frac{2}{5} \log 3 = 4 \log 3 + 1 + \log 3 - 2 - \frac{2}{5} \log 3 =$
 $= \frac{23}{5} \log 3 - 1 = 2,1942 - 1 = 1,1942$
11. $\log \sqrt[5]{0,04} + \log \sqrt[3]{\frac{0,25}{8}} + \log \sqrt{\frac{1,6}{5}}, \text{ si } \log 2 \approx 0,301$
 $\log \sqrt[5]{0,04} + \log \sqrt[3]{\frac{0,25}{8}} + \log \sqrt{\frac{1,6}{5}} =$
 $= \log \left(\frac{2^2}{100} \right)^{\frac{1}{5}} + \log \left(\frac{5^2}{2^3} \right)^{\frac{1}{3}} + \log \left(\frac{2^4}{5} \right)^{\frac{1}{2}} =$

$$\begin{aligned}
&= \frac{1}{5} \log \left(\frac{2^2}{100} \right) + \frac{1}{3} \log \left(\frac{\frac{5^2}{100}}{2^3} \right) + \frac{1}{2} \log \left(\frac{\frac{2^4}{10}}{5} \right) = \\
&= \frac{1}{5} (\log 2^2 - \log 10^2) + \frac{1}{3} \left[\log \left(\frac{5^2}{100} \right) - \log 2^3 \right] + \frac{1}{2} \left[\log \left(\frac{2^4}{10} \right) - \log 5 \right] = \\
&= \frac{1}{5} (2 \log 2 - 2 \log 10) + \frac{1}{3} \left[2 \log \frac{10}{2} - 2 \log 10 - 3 \log 2 \right] + \frac{1}{2} \left[4 \log 2 - \log 10 - \log \frac{10}{2} \right] = \\
&= \frac{1}{5} (2 \log 2 - 2) + \frac{1}{3} [2(1 - \log 2) - 2 - 3 \log 2] + \frac{1}{2} [4 \log 2 - 1 - (1 - \log 2)] = \\
&= \frac{2}{5} \log 2 - \frac{2}{5} + \frac{2}{3} - \frac{2}{3} \log 2 - \frac{2}{3} - \log 2 + 2 \log 2 - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} \log 2 = -\frac{7}{5} + \frac{7}{30} \log 2 = \\
&= -\frac{7}{5} + \frac{7}{30} \log 2 = -1,33
\end{aligned}$$

$$\begin{aligned}
12. \quad \log_a a^5 \sqrt{a} + \log_{\frac{1}{a}} \frac{\sqrt[3]{a}}{\sqrt{a}} &= \log_a a \cdot a^{\frac{1}{5}} + \log_{\frac{1}{a}} \sqrt[6]{\frac{a^2}{a^3}} = \log_a a^{\frac{6}{5}} + \log_{\frac{1}{a}} \left(\frac{1}{a} \right)^{\frac{1}{6}} = \\
&= \frac{6}{5} + \frac{1}{6} = \frac{41}{30}
\end{aligned}$$

$$\begin{aligned}
13. \quad \log_{a-b} \sqrt[3]{\frac{1}{a-b}} + \log_{\frac{a}{b}} \frac{b}{a} + \log_{a+b} \sqrt{a+b} &= \\
&= \log_{a-b} (a-b)^{-\frac{1}{3}} + \log_{\frac{a}{b}} \left(\frac{a}{b} \right)^{-1} + \log_{a+b} (a+b)^{\frac{1}{2}} = \\
&= \frac{1}{3} \log_{a-b} (a-b) - \log_{\frac{a}{b}} \left(\frac{a}{b} \right) + \frac{1}{2} \log_{a+b} (a+b) = \frac{1}{3} - 1 + \frac{1}{2} = -\frac{1}{6}
\end{aligned}$$

$$\begin{aligned}
14. \quad \log_a (\sqrt[3]{a} \cdot a^3) - \log_b (\sqrt[5]{b^2} : b^2) + \log_{a-b} (ab)^{-3} &= \\
&= \log_a \left(a^{\frac{1}{3}} \cdot a^3 \right) - \log_b \left(\sqrt[5]{b^{-8}} \right) - 3 = \log_a \left(a^{\frac{10}{3}} \right) - \log_b \left(b^{\frac{-8}{5}} \right) - 3 = \frac{10}{3} + \frac{8}{5} - 3 = \frac{29}{15}
\end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{\log_{a-b} \sqrt{\frac{1}{a-b}} + \log_{\frac{a}{b}} \frac{b}{a}}{\log_{a+b} \sqrt{a+b}} = \frac{\log_{a-b} (a-b)^{-\frac{1}{2}} + \log_{\frac{a}{b}} \left(\frac{a}{b}\right)^{-1}}{\log_{a+b} (a+b)^{\frac{1}{2}}} = \\
 & = \frac{-\frac{1}{2} \log_{a-b} (a-b) - \log_{\frac{a}{b}} \left(\frac{a}{b}\right)}{\frac{1}{2} \log_{a+b} (a+b)} = \frac{-\frac{1}{2} - 1}{\frac{1}{2}} = -3
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{\log_2 \sqrt[5]{8} + \log_2 16 + \log_2 \frac{1}{8}}{2 \log_2 4 - 3 \log_2 2} = \frac{\log_2 (2^3)^{\frac{1}{5}} + \log_2 2^4 + \log_2 2^{-3}}{\log_2 4^2 - \log_2 2^3} = \\
 & = \frac{\log_2 2^{\frac{3}{5}} + 4 \log_2 2 - 3 \log_2 2}{4 \log_2 2 - 3 \log_2 2} = \frac{\frac{3}{5} + 4 - 3}{4 - 3} = \frac{8}{5}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{\log_2 8 + \log_2 \frac{2}{25}}{\log_2 40 - \log_2 10} - \frac{\log_2 \frac{1}{5} - \log_2 \frac{25}{8}}{\log_2 2 + \log_2 4} = \\
 & = \frac{\log_2 2^3 + (\log_2 2 - \log_2 5^2)}{(\log_2 5 + \log_2 2^3) - (\log_2 5 + \log_2 2)} - \frac{\log_2 5^{-1} - (\log_2 5^2 - \log_2 2^3)}{\log_2 2 + \log_2 2^2} = \\
 & = \frac{3 + (1 - 2 \log_2 5)}{(\log_2 5 + 3) - (\log_2 5 + 1)} - \frac{-\log_2 5 - (2 \log_2 5 - 3)}{1 + 2} = \\
 & = \frac{4 - 2 \log_2 5}{2} - \frac{-3 \log_2 5 + 3}{3} = 2 - \log_2 5 - 1 + \log_2 5 = 1
 \end{aligned}$$

$$18. \quad \log_b \left(\frac{7 \cdot 2^3 \cdot 0,006^{-2}}{25 \cdot 3 \cdot 2^4} \right). \text{ Datos: } \begin{cases} \log_b 2 = 4 \\ \log_b 3 = 2 \\ \log_b 5 = -3 \end{cases}$$

$$\begin{aligned}
 & \log_b \left(\frac{7 \cdot 2^3 \cdot 0,006^{-2}}{25 \cdot 3 \cdot 2^4} \right) = \log_b (7 \cdot 2^3 \cdot 0,006^{-2}) - \log_b (25 \cdot 3 \cdot 2^4) = \\
 & = \log_b \left(\frac{2^2 \cdot 3^2}{5} \right)^3 + \log_b \left(\frac{3}{2^2 \cdot 5^3} \right)^{-2} - \left[\log_b (5^2) + \log_b \left(\frac{2^4}{5} \right) \right] = \\
 & = 3[2 \log_b 2 + 2 \log_b 3 - \log_b 5] - 2[\log_b 3 - 2 \log_b 2 - 3 \log_b 5] -
 \end{aligned}$$

$$\begin{aligned}
& -[2\log_b 5 + 4(4\log_b 2 - \log_b 5)] = \\
& = 3[2 \cdot 4 + 2 \cdot 2 - (-3)] - 2[2 - 2 \cdot 4 - 3(-3)] - \{2(-3) + 4[4 \cdot 4 - (-3)]\} = \\
& = 3[8 + 4 + 3] - 2[2 - 8 + 9] - \{-6 + 4[16 + 3]\} = -31
\end{aligned}$$

B.3 Hallar el término desconocido.

$$19. \log_a 125 = 3 \Rightarrow a^3 = 125 \Rightarrow a^3 = 5^3 \Rightarrow a = 5$$

$$20. \log_a 243 = 5 \Rightarrow a^5 = 243 \Rightarrow a^5 = 3^5 \Rightarrow a = 3$$

$$21. \log_{625} 25 = x \Rightarrow 625^x = 25 \Rightarrow 5^{4x} = 5^2 \Rightarrow 4x = 2 \Rightarrow x = \frac{1}{2}$$

$$22. \log_{32} 0,25 = x \Rightarrow 32^x = 0,25 \Rightarrow 2^{5x} = \frac{1}{4} \Rightarrow 2^{5x} = 2^{-2} \Rightarrow 5x = -2 \Rightarrow x = -\frac{2}{5}$$

$$23. \log_x 2 = \frac{1}{5} \Rightarrow x^{\frac{1}{5}} = 2 \Rightarrow \left(x^{\frac{1}{5}}\right)^5 = 2^5 \Rightarrow x^{\frac{5}{5}} = 2^5 \Rightarrow x = 32$$

B.4. Desarrollar expresiones logarítmicas:

$$24. \log_a \frac{x \cdot y}{z} = \log_a x \cdot y - \log_a z = \log_a x + \log_a y - \log_a z$$

$$25. \log_a \left(\frac{x}{y}\right)^2 = 2\log_a \frac{x}{y} = 2(\log_a x - \log_a y)$$

$$26. \log_a \frac{x \cdot y}{z} = \log_a x \cdot y - \log_a z = \log_a x + \log_a y - \log_a z$$

$$\begin{aligned}
27. \log_a \frac{x^3 y}{\sqrt{z}} &= \log_a x^3 y - \log_a \sqrt{z} = \log_a x^3 + \log_a y - \log_a z^{\frac{1}{2}} = \\
&= 3\log_a x + \log_a y - \frac{1}{2}\log_a z
\end{aligned}$$

B.5. Escribir como un solo logaritmo:

$$28. \quad \log(xy) - 2\log\left(\frac{x}{y}\right) = \log(xy) - \log\left(\frac{x}{y}\right)^2 = \log\left(\frac{xy}{\frac{x^2}{y^2}}\right) = \log\left(\frac{y^3}{x}\right)$$

$$29. \quad 2\ln(a-b) - \ln(a^2 - b^2) = \ln(a-b)^2 - \ln[(a+b)(a-b)] = \\ = \ln(a-b)^2 - \ln[(a+b)(a-b)] = \ln \frac{(a-b)^2}{(a+b)\cancel{(a-b)}} = \ln\left(\frac{a-b}{a+b}\right)$$

$$30. \quad 4\log_2 \frac{\sqrt{a-b}}{a} - \frac{1}{2}\log_2 \left(\frac{a-b}{a}\right)^4 = \log_2 \left(\frac{\sqrt{a-b}}{a}\right)^4 - \log_2 \left[\left(\frac{a-b}{a}\right)^4\right]^{\frac{1}{2}} = \\ = \log_2 \frac{(a-b)^2}{a^4} - \log_2 \left(\frac{a-b}{a}\right)^2 = \log_2 \left[\frac{\frac{(a-b)^2}{a^4}}{\frac{(a-b)^2}{a^2}}\right] = \log_2 \left[\frac{a^2(a-b)^2}{a^4(a-b)^2}\right] = \\ \log_2 \left(\frac{1}{a^2}\right) = \log_2(a^{-2})$$

$$31. \quad 2\log_5(x) - \frac{1}{3}\log_5(b) + (x+2)\log_5(7) = \log_5 x^2 - \log_5 b^{\frac{1}{3}} + \log_5 7^{x+2} = \\ = \log_5 \frac{x^2}{b^{\frac{1}{3}}} + \log_5 7^{x+2} = \log_5 \frac{x^2 \cdot 7^{x+2}}{b^{\frac{1}{3}}} = \log_5 \frac{x^2 \cdot 7^{x+2}}{\sqrt[3]{b}}$$

$$32. \quad \log\left(\frac{a}{b}\right) + \log\left(\frac{b}{c}\right) + \log\left(\frac{c}{d}\right) - \log\left(\frac{ay}{xd}\right) = \log\left(\frac{a}{b} \cdot \frac{b}{c} \cdot \frac{c}{d}\right) - \log\left(\frac{ay}{xd}\right) = \\ = \log\left(\frac{a}{cd}\right) - \log\left(\frac{ay}{xd}\right) = \log\left(\frac{\frac{a}{cd}}{\frac{ay}{xd}}\right) = \log\left(\frac{\frac{a}{cd}}{\frac{ay}{xd}}\right) = \log\left(\frac{x}{cy}\right)$$

$$\begin{aligned} 33. \quad & \log_2(xy) - \log_2\left(\frac{x}{y^2}\right) + \frac{1}{2}\log_2\left(\frac{x^2y}{2}\right) = \\ & = \log_2\left(\frac{xy}{\frac{x}{y^2}}\right) + \log_2\left(\frac{x^2y}{2}\right) = \log_2\left(y^3\left(\frac{x^2y}{2}\right)\right) = \log_2\left(\frac{x^2y^4}{2}\right) \end{aligned}$$

TEMAS RELACIONADOS

- Ecuaciones logarítmicas.
- Ecuaciones exponenciales.
