

## Solución a los ejercicios del tema Límites de sucesiones

### Ejercicio 1

Calcula los siguientes límites:

$$a) \lim_{n \rightarrow \infty} (7 + n) = \boxed{\infty}$$

$$b) \lim_{n \rightarrow \infty} \left( 7 - \frac{1}{n} \right) = \boxed{7}$$

$$c) \lim_{n \rightarrow \infty} (7 - n^2) = \boxed{-\infty}$$

$$d) \lim_{n \rightarrow \infty} \left( 6 - \frac{1}{n^3} \right) = \boxed{6}$$

$$e) \lim_{n \rightarrow \infty} 7n = \boxed{\infty}$$

$$f) \lim_{n \rightarrow \infty} \left( \frac{3}{n} \cdot n \right) = \boxed{3}$$

$$g) \lim_{n \rightarrow \infty} \left( \frac{1}{3} \right)^n = \boxed{0}$$

$$h) \lim_{n \rightarrow \infty} 3^n = \boxed{\infty}$$

$$i) \lim_{n \rightarrow \infty} \left[ -5n^3 (n^2 - 100) \right] = \boxed{-\infty}$$

$$j) \lim_{n \rightarrow \infty} (2n^3)^{-2} = \boxed{0}$$

$$k) \lim_{n \rightarrow \infty} (23 + 10^{-n}) = \boxed{23}$$

$$l) \lim_{n \rightarrow \infty} (8n^{-2} - 7n^{-3} - 500) = \boxed{-500}$$

### Ejercicio 2

Estudia hacia qué tenderán las siguientes sucesiones cuando la  $n$  es muy grande:

$$a) \quad a_n = \frac{n}{n^5} \qquad \lim_{n \rightarrow \infty} \frac{n}{n^5} = \lim_{n \rightarrow \infty} \frac{1}{n^4} = \boxed{0}$$

$$b) \quad a_n = \frac{n^2 + 1}{n^5} \qquad \lim_{n \rightarrow \infty} \frac{n^2 + 1}{n^5} = \boxed{0}$$

$$c) \quad a_n = n^7 + n^5 \qquad \lim_{n \rightarrow \infty} n^7 + n^5 = \boxed{\infty}$$

$$d) \quad a_n = \frac{n^8 - 4n^3 - 5n^2}{1000 + n^2} \qquad \lim_{n \rightarrow \infty} \frac{n^8 - 4n^3 - 5n^2}{1000 + n^2} = \boxed{\infty}$$

$$e) \quad a_n = -2^n \qquad \lim_{n \rightarrow \infty} -2^n = \boxed{-\infty}$$

### Ejercicio 3

Calcula:

$$a) \lim_{n \rightarrow \infty} \frac{2n^2 - 5n + 7}{3n^2} = \boxed{\frac{2}{3}}$$

$$b) \lim_{n \rightarrow \infty} \frac{5n^4 - 2n^3 + n^2 - n + 1}{3n^3 + 2n^2 - n + 3} = \boxed{\infty}$$

$$c) \lim_{n \rightarrow \infty} \frac{3n^2 - 1}{4n^3 + 2} = \boxed{0}$$

$$d) \lim_{n \rightarrow \infty} \frac{(n+1)^2}{2n^2} = \lim_{n \rightarrow \infty} \frac{n^2 + 2n + 1}{2n^2} = \boxed{\frac{1}{2}}$$

$$e) \lim_{n \rightarrow \infty} \frac{(n+1)^2 - (n-1)^2}{5n+3} = \lim_{n \rightarrow \infty} \frac{(n+1+n-1)(n+1-(n-1))}{5n+3} = \lim_{n \rightarrow \infty} \frac{2n \cdot 2}{5n+3} = \boxed{\frac{4}{5}}$$

$$f) \lim_{n \rightarrow \infty} \frac{(n+1)^2 + (n-1)^2}{5n+3} = \lim_{n \rightarrow \infty} \frac{n^2 + 2n + 1 + n^2 - 2n + 1}{5n+3} = \lim_{n \rightarrow \infty} \frac{2n^2 + 2}{5n+3} = \boxed{\infty}$$

$$g) \lim_{n \rightarrow \infty} \frac{n+10}{n} = \boxed{1}$$

### Ejercicio 4

La sucesión  $a_n = \frac{1}{4n}$  tiene por límite 0, y la sucesión  $b_n = 2n+1$  tiende a  $+\infty$ . Estudia si la sucesión  $c_n = a_n \cdot b_n$  es convergente:

$$\lim_{n \rightarrow \infty} (a_n \cdot b_n) = \lim_{n \rightarrow \infty} \frac{2n+1}{4n} = \boxed{\frac{1}{2}}$$

### Ejercicio 5

Calcula el límite de las siguientes sucesiones aplicando propiedades:

$$a) \lim_{n \rightarrow \infty} \left( 3 + \frac{2}{n} \right) = \boxed{\lim_{n \rightarrow \infty} 3 + \frac{\lim_{n \rightarrow \infty} 2}{\lim_{n \rightarrow \infty} n} = 3 + \frac{2}{\infty} = 3 + 0 = 3}$$

$$b) \lim_{n \rightarrow \infty} 2^{\frac{1}{n}} = \lim_{n \rightarrow \infty} 2^{\lim_{n \rightarrow \infty} \frac{1}{n}} = 2^{\lim_{n \rightarrow \infty} \frac{1}{n}} = 2^0 = \boxed{1}$$

## Ejercicio 6

Estos límites presentan una indeterminación; averíguala y resuélvela:

$$a) \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+7}\right)^n \underset{\substack{\text{IND } 1^\infty \\ \Downarrow}}{=} e^{\lim_{n \rightarrow \infty} n \cdot \left(1 + \frac{1}{n+7} - 1\right)} = e^{\lim_{n \rightarrow \infty} \frac{n}{n+7}} = e^1 = \boxed{e}$$

$$b) \lim_{n \rightarrow \infty} \frac{2n}{\sqrt{n^2 + 3n - 2}} \underset{\substack{\text{IND } \frac{\infty}{\infty} \\ \Downarrow}}{=} = \lim_{n \rightarrow \infty} \frac{\frac{2n}{n}}{\sqrt{\frac{n^2}{n^2} + \frac{3n}{n^2} - \frac{2}{n^2}}} = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{1 + \frac{3}{n} - \frac{2}{n^2}}} = \frac{2}{\sqrt{1}} = \boxed{2}$$

$$c) \lim_{n \rightarrow \infty} \sqrt{n+1} - \sqrt{n} \underset{\substack{\text{IND } (\infty - \infty) \\ \Downarrow}}{=} \lim_{n \rightarrow \infty} \frac{(\sqrt{n+1} - \sqrt{n})(\sqrt{n+1} + \sqrt{n})}{(\sqrt{n+1} + \sqrt{n})} = \lim_{n \rightarrow \infty} \frac{(n+1) - n}{\sqrt{n+1} + \sqrt{n}} =$$

$$= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\infty} = \boxed{0}$$

d)  $\lim_{n \rightarrow \infty} (n^2 - n) =$  (En este límite lo difícil es no hacerlo de forma intuitiva. Debemos averiguar cuál es la indeterminación y, luego, resolverla. Como el problema está en la resta, al sacar factor común, desaparece la indeterminación)

$$\lim_{n \rightarrow \infty} (n^2 - n) \underset{\substack{\text{IND } \infty - \infty \\ \Downarrow}}{=} \lim_{n \rightarrow \infty} n(n-1) = \infty \cdot \infty = \boxed{\infty}$$

$$e) \lim_{n \rightarrow \infty} \frac{n + \sqrt{9n^2 + 2}}{3n - 3} \underset{\substack{\text{IND } \frac{\infty}{\infty} \\ \Downarrow}}{=} \lim_{n \rightarrow \infty} \frac{\frac{n}{n} + \sqrt{\frac{9n^2}{n^2} + \frac{2}{n^2}}}{\frac{3n}{n} - \frac{3}{n}} = \lim_{n \rightarrow \infty} \frac{1 + \sqrt{9 + \frac{2}{n^2}}}{3 - \frac{3}{n}} = \frac{1+3}{3} = \boxed{\frac{4}{3}}$$

$$f) \lim_{n \rightarrow \infty} \left(\frac{n+4}{n+3}\right)^{n+2} \underset{\substack{\text{IND } 1^\infty \\ \Downarrow}}{=} e^{\lim_{n \rightarrow \infty} (n+2) \cdot \left(\frac{n+4}{n+3} - 1\right)} = e^{\lim_{n \rightarrow \infty} (n+2) \cdot \left(\frac{n+3}{n+3} + \frac{1}{n+3} - 1\right)} = e^{\lim_{n \rightarrow \infty} \frac{n+2}{n+3}} = \boxed{e}$$

$$g) \lim_{n \rightarrow \infty} \sqrt{n^2 + n} - n \underset{\substack{\text{IND } \infty - \infty \\ \Downarrow}}{=} \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2 + n} - n)(\sqrt{n^2 + n} + n)}{(\sqrt{n^2 + n} + n)} =$$

$$= \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2 + n} - n)(\sqrt{n^2 + n} + n)}{(\sqrt{n^2 + n} + n)} = \lim_{n \rightarrow \infty} \frac{n^2 + n - n^2}{\sqrt{n^2 + n} + n} = \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 + n} + n} \underset{\substack{\text{IND } \frac{\infty}{\infty} \\ \Downarrow}}{=}$$

$$= \lim_{n \rightarrow \infty} \frac{\frac{n}{n}}{\sqrt{\frac{n^2}{n^2} + \frac{n}{n^2} + \frac{n}{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{n} + 1}} = \frac{1}{\sqrt{1+1}} = \boxed{\frac{1}{2}}$$

$$\begin{aligned} h) \quad \lim_{n \rightarrow \infty} \left( \frac{5n-4}{5n+2} \right)^{\frac{n+1}{3}} &\stackrel{\text{IND } 1^\infty}{=} e^{\lim_{n \rightarrow \infty} \left( \frac{n+1}{3} \right) \left( \frac{5n-4}{5n+2} - 1 \right)} = e^{\lim_{n \rightarrow \infty} \left( \frac{n+1}{3} \right) \left( \frac{5n+2}{5n+2} - \frac{6}{5n+2} - 1 \right)} = e^{\lim_{n \rightarrow \infty} \frac{-6(n+1)}{3(5n+2)}} = \\ &= e^{\lim_{n \rightarrow \infty} \frac{-2n-2}{5n+2}} = e^{\lim_{n \rightarrow \infty} \frac{-2n-2}{5n+2}} = \boxed{e^{\frac{-2}{5}} = \sqrt[5]{\frac{1}{e^2}}} \end{aligned}$$

### Ejercicio 7

Calcula los siguientes límites:

$$\begin{aligned} a) \quad \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^{n+3} &= \lim_{n \rightarrow \infty} \left[ \left( 1 + \frac{1}{n} \right)^n \cdot \left( 1 + \frac{1}{n} \right)^3 \right] = \\ &= \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n \cdot \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^3 = e \cdot 1^3 = \boxed{e} \end{aligned}$$

$$\begin{aligned} b) \quad \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^{-2n} &= \lim_{n \rightarrow \infty} \left[ \left( 1 + \frac{1}{n} \right)^n \right]^{-2} = \lim_{n \rightarrow \infty} \left[ \left( 1 + \frac{1}{n} \right)^n \right]^{-2} = \\ &= \left[ \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n \right]^{\lim_{n \rightarrow \infty} -2} = e^{\lim_{n \rightarrow \infty} -2} = \boxed{e^{-2} = \frac{1}{e^2}} \end{aligned}$$

### Ejercicio 8

Calcula los siguientes límites:

$$a) \quad \lim_{x \rightarrow 1} (x^2 - 5x + 3) = 1 - 5 + 3 = \boxed{-1}$$

$$b) \quad \lim_{x \rightarrow +\infty} (x^7 - 8x^2 + 3x) = \boxed{+\infty} \quad (\text{porque el término de mayor grado es positivo})$$

(Sigue →)

(Continuación)

$$c) \lim_{x \rightarrow -\infty} (x^2 + 3x^3 + 1) = \boxed{-\infty} \text{ (porque el término de mayor grado es negativo)}$$

$$d) \lim_{x \rightarrow 0} \frac{3x^4}{x^3 + x^2} \underset{\substack{\text{IND} \frac{0}{0}}}{=} \lim_{x \rightarrow 0} \frac{3x^2 \cdot x^2}{x^2(x+1)} = \lim_{x \rightarrow 0} \frac{3x^2}{(x+1)} = \frac{0}{0+1} = \boxed{0}$$

$$e) \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} \underset{\substack{\text{IND} \frac{0}{0}}}{=} \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 1)}{(x+1)(x-1)} = \lim_{x \rightarrow 1} \frac{x^2 + x + 1}{x+1} = \boxed{\frac{3}{2}}$$

$$f) \lim_{x \rightarrow 5} \frac{x^2 - 25}{x^2 - 5x} \underset{\substack{\text{IND} \frac{0}{0}}}{=} \lim_{x \rightarrow 5} \frac{(x+5)(x-5)}{x(x-5)} = \lim_{x \rightarrow 5} \frac{x+5}{x} = \boxed{2}$$

$$g) \lim_{x \rightarrow \sqrt{5}} \frac{x - \sqrt{5}}{x^2 - 5} \underset{\substack{\text{IND} \frac{0}{0}}}{=} \lim_{x \rightarrow \sqrt{5}} \frac{x - \sqrt{5}}{(x + \sqrt{5})(x - \sqrt{5})} = \lim_{x \rightarrow \sqrt{5}} \frac{1}{x + \sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}}$$

$$\begin{aligned} h) \lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x-1} &\underset{\substack{\text{IND} \frac{0}{0}}}{=} \lim_{x \rightarrow 1} \frac{(\sqrt{x+3} - 2) \cdot (\sqrt{x+3} + 2)}{(x-1)(\sqrt{x+3} + 2)} = \lim_{x \rightarrow 1} \frac{x+3-4}{(x-1)(\sqrt{x+3} + 2)} = \\ &= \lim_{x \rightarrow 1} \frac{x-1}{(x-1)(\sqrt{x+3} + 2)} = \lim_{x \rightarrow 1} \frac{1}{\sqrt{x+3} + 2} = \frac{1}{\sqrt{4} + 2} = \boxed{\frac{1}{4}} \end{aligned}$$

$$i) \lim_{x \rightarrow +\infty} \left( \frac{4x+1}{2x} \right)^x = 2^\infty = \boxed{\infty}$$

### Ejercicio 9

Calcula los siguientes límites:

$$a) \quad \lim_{x \rightarrow 3} \frac{x+1}{x-1} = \frac{4}{2} = \boxed{2} \quad \text{No hay indeterminación}$$

$$b) \quad \lim_{x \rightarrow \infty} \frac{x+1}{x-1} \underset{\substack{\text{IND} \\ \frac{\infty}{\infty}}}{=} \lim_{x \rightarrow \infty} \frac{\frac{x}{x} + \frac{1}{x}}{\frac{x}{x} - \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} = \frac{1+0}{1-0} = \boxed{1}$$

$$c) \quad \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} \underset{\substack{\text{IND} \\ \frac{0}{0}}}{=} \lim_{x \rightarrow 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x \rightarrow 1} (x+1) = \boxed{2}$$

$$d) \quad \lim_{x \rightarrow 0} \frac{x}{1 - \sqrt{1-x}} \underset{\substack{\text{IND} \\ \frac{0}{0}}}{=} \lim_{x \rightarrow 0} \frac{x(1 + \sqrt{1-x})}{(1 - \sqrt{1-x})(1 + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{x(1 + \sqrt{1-x})}{1 - (1-x)} =$$

$$= \lim_{x \rightarrow 0} \frac{x(1 + \sqrt{1-x})}{x} = \lim_{x \rightarrow 0} (1 + \sqrt{1-x}) = \boxed{2}$$

$$e) \quad \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + x}}{x} \underset{\substack{\text{IND} \\ \frac{\infty}{\infty}}}{=} \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{x^2 + x}{x^2}}}{\frac{x}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{x}}}{1} = \sqrt{1+0} = \boxed{1}$$

$$f) \quad \lim_{x \rightarrow 3} \frac{\sqrt{x+6} - 3}{x - 3} \underset{\substack{\text{IND} \\ \frac{0}{0}}}{=} \lim_{x \rightarrow 3} \frac{(\sqrt{x+6} - 3)(\sqrt{x+6} + 3)}{(x-3)(\sqrt{x+6} + 3)} = \lim_{x \rightarrow 3} \frac{x+6-9}{(x-3)(\sqrt{x+6} + 3)} =$$

$$= \lim_{x \rightarrow 3} \frac{x-3}{(x-3)(\sqrt{x+6} + 3)} = \lim_{x \rightarrow 3} \frac{1}{\sqrt{x+6} + 3} = \frac{1}{\sqrt{9+3}} = \boxed{\frac{1}{6}}$$